# Hw 3 Solution

## Problem 2.2.2 Solution

From Example 2.5, we can write the PMF of $X$ and the PMF of $R$ as

$$P_X(x) = \begin{cases} 1/8 & x = 0 \\ 3/8 & x = 1 \\ 3/8 & x = 2 \\ 1/8 & x = 3 \\ 0 & \text{otherwise} \end{cases} \qquad P_R(r) = \begin{cases} 1/4 & r = 0 \\ 3/4 & r = 2 \\ 0 & \text{otherwise} \end{cases}$$

From the PMFs $P_X(x)$ and $P_R(r)$, we can calculate the requested probabilities

(a) $P[X = 0] = P_X(0) = 1/8$.

(b) $P[X < 3] = P_X(0) + P_X(1) + P_X(2) = 7/8$.

(c) $P[R > 1] = P_R(2) = 3/4$.

## Problem 2.2.4 Solution

(a) We choose $c$ so that the PMF sums to one.

$$\sum_x P_X(x) = \frac{c}{2} + \frac{c}{4} + \frac{c}{8} = \frac{7c}{8} = 1$$

Thus $c = 8/7$.

(b)

$$P[X = 4] = P_X(4) = \frac{8}{7 \cdot 4} = \frac{2}{7}$$

(c)

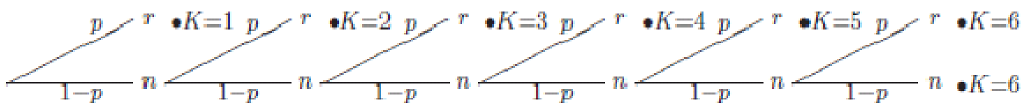
$$P[X < 4] = P_X(2) = \frac{8}{7 \cdot 2} = \frac{4}{7}$$

(d)

$$P[3 \leq X \leq 9] = P_X(4) + P_X(8) = \frac{8}{7 \cdot 4} + \frac{8}{7 \cdot 8} = \frac{3}{7}$$

## Problem 2.2.9 Solution

(a) In the setup of a mobile call, the phone will send the "SETUP" message up to six times. Each time the setup message is sent, we have a Bernoulli trial with success probability $p$. Of course, the phone stops trying as soon as there is a success. Using $r$ to denote a successful response, and $n$ a non-response, the sample tree is




(b) We can write the PMF of $K$, the number of "SETUP" messages sent as

$$P_K(k) = \begin{cases} (1-p)^{k-1}p & k = 1, 2, \ldots, 5 \\ (1-p)^5 p + (1-p)^6 = (1-p)^5 & k = 6 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Note that the expression for $P_K(6)$ is different because $K = 6$ if either there was a success or a failure on the sixth attempt. In fact, $K = 6$ whenever there were failures on the first five attempts which is why $P_K(6)$ simplifies to $(1-p)^5$.

(c) Let $B$ denote the event that a busy signal is given after six failed setup attempts. The probability of six consecutive failures is $P[B] = (1-p)^6$.

(d) To be sure that $P[B] \leq 0.02$, we need $p \geq 1 - (0.02)^{1/6} = 0.479$.

## Problem 2.3.3 Solution

Whether a hook catches a fish is an independent trial with success probability $h$. The the number of fish hooked, $K$, has the binomial PMF

$$P_K(k) = \begin{cases} \binom{m}{k} h^k (1-h)^{m-k} & k = 0, 1, \ldots, m \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

## Problem 2.3.7 Solution

Since an average of $T/5$ buses arrive in an interval of $T$ minutes, buses arrive at the bus stop at a rate of $1/5$ buses per minute.

(a) From the definition of the Poisson PMF, the PMF of $B$, the number of buses in $T$ minutes, is

$$P_B(b) = \begin{cases} (T/5)^b e^{-T/5}/b! & b = 0, 1, \ldots \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

(b) Choosing $T = 2$ minutes, the probability that three buses arrive in a two minute interval is

$$P_B(3) = (2/5)^3 e^{-2/5}/3! \approx 0.0072 \tag{2}$$

(c) By choosing $T = 10$ minutes, the probability of zero buses arriving in a ten minute interval is

$$P_B(0) = e^{-10/5}/0! = e^{-2} \approx 0.135 \tag{3}$$

(d) The probability that at least one bus arrives in $T$ minutes is

$$P[B \geq 1] = 1 - P[B = 0] = 1 - e^{-T/5} \geq 0.99 \tag{4}$$

Rearranging yields $T \geq 5 \ln 100 \approx 23.0$ minutes.

## Problem 2.3.11 Solution

The packets are delay sensitive and can only be retransmitted $d$ times. For $t < d$, a packet is transmitted $t$ times if the first $t-1$ attempts fail followed by a successful transmission on attempt $t$. Further, the packet is transmitted $d$ times if there are failures on the first $d-1$ transmissions, no matter what the outcome of attempt $d$. So the random variable $T$, the number of times that a packet is transmitted, can be represented by the following PMF.

$$P_T(t) = \begin{cases} p(1-p)^{t-1} & t = 1, 2, \ldots, d-1 \\ (1-p)^{d-1} & t = d \\ 0 & \text{otherwise} \end{cases} \tag{1}$$